

# Meta-Analysis of $d$

In Field (2001) I describe the basis of meta-analysis and the difference between fixed and random-effects methods. I also describes the procedure for these tests when correlation coefficients are used as the effect size metric. However, another popular metric is the effect size estimate  $d$ , which is traditionally used for expressing the magnitude of differences between groups (see Cohen, 1988). This handout describes meta-analytic procedures for this effect size.

## Hedges and Olkin's Fixed-Effect Method

The definitive description of this technique can be found in Hedges and Olkin's (1985) text. Hedges and Olkin's method extended Glass' (1976) earlier work in which he proposed an estimate of effect size based on the difference between group means standardized using the standard deviation of the control group. Hedges and Olkin's method is essentially the same except that a pooled variance estimate is used to standardise the difference between group means. They called this effect-size measure,  $g$ . As such, this method is usually applied to effect-size measures based on differences ( $d$ ). Effect sizes are interchangeable, and for example, an effect size  $d$  can be converted into a correlation coefficient and vice versa. Equation (1) demonstrates how this conversion (and the conversion back) is achieved.

$$d = \frac{2r}{\sqrt{1-r^2}} \qquad r = \frac{d}{\sqrt{d^2+4}} \qquad (1)$$

Having established the effect size estimate, Hedges and Olkin suggest a correction that produces an unbiased effect size estimate. This correction is shown in equation (2), in which  $N$  represents the total sample size on which  $d$  is based (see p. 79-81 in Hedges and Olkin 1985).

$$d_{unbiased} = \left(1 - \frac{3}{4(N-2)-1}\right) \times d \qquad (2)$$

Having established the unbiased effect size estimate, the average effect size ( $d_+$ ) can be calculated using a weighted average based on the variance of these unbiased effect sizes ( $\sigma^2_{(d)}$ ). The variance of effect sizes is calculated using equation (3) (see p. 86 of Hedges and Olkin, 1985) in which the  $n$ s refer to the sample size of two experimental groups. The resulting variance estimate is placed into equation (4) (see p. 111 of Hedges and Olkin, 1985) to obtain the weighted average effect size ( $d_i$  is the unbiased effect size for study  $i$ ).

$$\hat{\sigma}^2_{d_i} = \frac{n_i^e + n_i^c}{n_i^e n_i^c} + \frac{d_i^2}{2(n_i^e + n_i^c)} \qquad (3)$$

$$d_+ = \frac{\sum_{i=1}^k \frac{d_i}{\hat{\sigma}^2_{d_i}}}{\sum_{i=1}^k \frac{1}{\hat{\sigma}^2_{d_i}}} \qquad (4)$$

To obtain a standardised value of the mean (a  $Z$  score), the mean effect size is simply divided by an estimate of the standard deviation of effect sizes (see equation (6)). The standard deviation of the mean is given in equation (5) (based on the square root of the equation of the variance given on p. 112 of Hedges and Olkin, 1985).

$$\hat{\sigma}_{d_+} = \sqrt{\left( \sum_{i=1}^k \frac{1}{\hat{\sigma}_{d_i}^2} \right)^{-1}} \quad (5)$$

$$Z = \frac{d_+}{\hat{\sigma}_{d_+}} \quad (6)$$

Finally, to obtain a test of the homogeneity of effect size, the statistic  $Q$  is used, which has a chi-square distribution with  $k-1$  degrees of freedom (where  $k$  is the number of studies being assimilated). Equation (7) shows how this test is calculated: it is the standardised sum of squared differences between each effect size and the mean effect size.

$$Q = \sum_{i=1}^k \frac{(d_i - d_+)^2}{\hat{\sigma}_{d_i}^2} \quad (7)$$

## Further Reading

- Field, A. P. (2001). Meta-analysis of correlation coefficients: a Monte Carlo comparison of fixed- and random-effects methods. *Psychological Methods*, 6 (2), 161–180.
- Hedges, L. V., & Olkin, I. (1985). *Statistical methods for meta-analysis*. Orlando, FL: Academic Press.
- Hedges, L. V. & Vevea, J. L. (1998). Fixed- and random-effects models in meta-analysis. *Psychological Methods*, 3, 486-504.
- Johnson, B. T., Mullen, B., & Salas, E. (1995). Comparison of three major meta-analytic approaches. *Journal of Applied Psychology*, 80 (1), 94-106.
- Rosenthal, R. (1991). *Meta-analytic procedures for social research* (revised). Newbury Park, CA: Sage.
- Wolf, F. M. (1986). *Meta-Analysis*. Sage university paper series on quantitative applications in the social sciences, 07–061. Newbury Park, CA: Sage.